Problem 1.8

Determine the sum of two vectors $5e^{i\pi/6}$ and $4e^{i\pi/3}$ and find the angle between the resultant and the first vector.

Solution

Let $z_1 = 5e^{i\pi/6}$ and $z_2 = 4e^{i\pi/3}$.

$$\begin{aligned} z_1 + z_2 &= 5e^{i\pi/6} + 4e^{i\pi/3} \\ &= 5\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) + 4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) \\ &= 5\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) + 4\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) \\ &= \left(\frac{5\sqrt{3}}{2} + 2\right) + \left(i\frac{5}{2} + 2i\sqrt{3}\right) \\ &= \frac{5\sqrt{3}+4}{2} + i\frac{5+4\sqrt{3}}{2} \end{aligned}$$

Find the angle θ that $z_1 + z_2$ makes with the positive real axis.

$$\theta = \tan^{-1}\left(\frac{\frac{5+4\sqrt{3}}{2}}{\frac{5\sqrt{3}+4}{2}}\right) = \tan^{-1}\left(\frac{5+4\sqrt{3}}{5\sqrt{3}+4}\right) \approx 0.7556 \approx 43.29^\circ$$

Therefore, the angle between the resultant and the first vector is

$$\theta - \frac{\pi}{6} \approx 43.29^\circ - 30^\circ = 13.29^\circ.$$